

Logarithmic correction in the deformed AdS_5 model to produce the heavy quark potential and QCD beta function

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ABSTRACT: We study the *holographic* QCD model which contains a quadratic term $-\sigma z^2$ and a logarithmic term $-c_0 \log[(z_{IR} - z)/z_{IR}]$ with an explicit infrared cut-off z_{IR} in the deformed AdS_5 warp factor. We investigate the heavy quark potential for three cases, i.e, with only quadratic correction, with both quadratic and logarithmic corrections and with only logarithmic correction. We solve the dilaton field and dilation potential from the Einstein equation, and investigate the corresponding beta function in the Gürsoy -Kiritsis-Nitti (GKN) framework. Our studies show that in the case with only quadratic correction, a negative σ or the Andreev-Zakharov model is favored to fit the heavy quark potential and to produce the QCD beta-function at 2-loop level, however, the dilaton potential is unbounded in infrared regime. One interesting observing for the case of positive σ , or the soft-wall AdS_5 model is that the corresponding beta-function exists an infrared fixed point. In the case with only logarithmic correction, the heavy quark Cornell potential can be fitted very well, the corresponding beta-function agrees with the QCD beta-function at 2-loop level reasonably well, and the dilaton potential is bounded from below in infrared. At the end, we propose a more compact model which has only logarithmic correction in the deformed warp factor and has less free parameters.

KEYWORDS: AdS/CFT, holography, heavy quark potential, QCD beta function.

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1. Introduction

Quantum Chromodynamics (QCD) has been accepted as the basic theory of describing strong interaction for more than 30 years. However, it is still a challenge to solve QCD in non-perturbative region where gauge interaction is strong. Recently, the conjecture of the gravity/gauge duality [1] has revived the hope of understanding QCD in strongly coupled region using string theory. The AdS/CFT duality has been widely used to discuss the meson spectra [2, 3, 4] and dense and hot quark matter [5, 6, 7, 8, 9]. The string description of realistic QCD has not been successfully formulated yet. Many efforts are invested in searching for such a realistic description by using the "top-down" approach, *i.e.* by deriving holographic QCD from string theory

[10, 11, 12, 13], as well as by using the "bottom-up" approach, *i.e.* by examining possible *holographic* QCD models from experimental data and lattice results.

In the "bottom-up" approach, the most economic way might be to search for a deformed AdS_5 metric [14, 15, 16, 17, 18, 19, 20, 21], which can describe the known experimental data and lattice results of QCD, e.g. hadron spectra and the heavy quark potential. The simplest holographic QCD model is the hard-wall AdS_5 model [22, 2], which can describe the lightest meson spectra in 80 – 90% agreement with the experimental data. However, the hard-wall model cannot produce the Regge behavior for higher excitations. It is regarded that the Regge behavior is related to the linear confinement. It has been suggested in Ref. [14] that a negative quadratic dilaton term $-z^2$ in the action is needed to produce the right linear Regge behavior of ρ mesons or the linear confinement.

The most direct physical quantity related to the confinement is the heavy-quark potential. The lattice result which is consistent with the so called Cornell potential [23] has the form of

$$V_{Q\bar{Q}}(R) = -\frac{\kappa}{R} + \sigma_{str}R + V_0. \quad (1.1)$$

Where $\kappa \approx 0.48$, $\sigma_{str} \approx 0.183\text{GeV}^2$ and $V_0 = -0.25\text{GeV}$, the first two parameters can be interpreted as $\frac{4\alpha_s}{3}$ and QCD "string" tension, respectively.

In order to produce linear behavior of heavy flavor potential, Andreev and Zakharov in Ref.[15] suggested a positive quadratic term modification [16] in the deformed warp factor of the metric, which is different from the soft-wall model in [14]. Andreev-Zakharov model has been further studied in many other articles [24, 25, 26, 27]. In Ref. [27], the authors found that the heavy quark potential from the positive quadratic model is closer to the Cornell potential than that from the backreaction model [17], which contains higher order corrections.

It is clearly seen from the Cornell potential that the Coulomb potential dominates in the ultraviolet (UV) region and the linear potential dominates in the infrared (IR) region. It motivates people to take into account the QCD running coupling effect into the modified metric [19, 20, 21]. In Ref.[21], Pirner and Galow have proposed a deformed metric which resembles the QCD running coupling, and the Pirner-Galow metric can fit the Cornell potential reasonably well. However, as shown in Ref. [28] the corresponding dilaton potential solved from the Einstein equation is unstable, and the corresponding beta function does not agree with the QCD beta function.

The motivation of this paper is to search for a deformed AdS_5 metric, which can describe the heavy quark potential as well as the QCD β function and at the same time can have a stable dilaton potential from the gravity side. In [13], we have proposed the soft-wall Dp-Dq model, which contains a quadratic correction and a logarithmic correction $c_0 \log z$. The logarithmic dilaton correction is derived from the top-down method, which is general for Dp-Dq system except $p = 3$. As pointed in [19], that the logarithmic term $c_0 \log z$ itself cannot produce confinement, while

a logarithmic correction with an infrared cut-off in the form of $c_0 \log(z_{IR} - z)$ can have confinement at IR. Therefore, we propose a *holographic* QCD model, which contains a quadratic term $-\sigma z^2$ and a logarithmic term $-c_0 \log[(z_{IR} - z)/z_{IR}]$ with an explicit infrared cut-off z_{IR} in the deformed AdS_5 warp factor, where we assume σ, c_0 can be either positive or negative. This model is found to have the same metric structure of the Pirner-Galow's model [21] in UV and IR region respectively. We investigate the heavy quark potential in the proposed model for three cases, i.e, with only quadratic correction, with both quadratic and logarithmic corrections and with only logarithmic correction. We solve the dilaton field and dilation potential from the Einstein equation, and investigate the corresponding beta function in the Gürsoy-Kiritsis-Nitti (GKN) [19] framework. Our studies show that in the case with only quadratic correction, the produced heavy quark potential has both Coulomb part and linear part and fits the Cornell potential qualitatively well. In the case with only logarithmic correction, the heavy quark Cornell potential can be perfectly fitted and the corresponding beta-function agrees with the QCD beta-function reasonably well. At the end, we propose a more compact model which has only logarithmic correction in the deformed warp factor and has less free parameters.

The paper is organized as follows. In section II, we derive the general formulae for heavy quark potential in the framework of AdS/CFT, and introduce the GKN framework to construct the gravity dual of the 5D holographic model, and calculate the β function. In section III, we construct our *holographic* QCD model, which contains a quadratic term $-\sigma z^2$ and a logarithmic term $-c_0 \log[(z_{IR} - z)/z_{IR}]$ with an explicit infrared cut-off z_{IR} in the deformed AdS_5 warp factor. We fit the heavy quark potential in this model for three cases, i.e, with only quadratic correction, with both quadratic and logarithmic corrections and with only logarithmic correction. We solve the dilaton field and dilation potential from the Einstein equation, and investigate the corresponding beta function. In section IV, we propose a more compact model with only logarithmic correction and with less parameters. The summary and discussion is given in section V.

2. The formalism

The AdS_5 metric in the Euclidean space takes the form of

$$ds^2 = G_{\mu\nu}^s dX^\mu dX^\nu = \frac{L^2}{z^2} (dt^2 + d\vec{x}^2 + dz^2), \quad (2.1)$$

where $G_{\mu\nu}^s$ indicates the metric in the string frame, and L is the radius of AdS_5 . To search for the possible *holographic* QCD models, the most economic way of breaking conformal invariance is to add a deformed warp factor $h(z)$ in the metric background,

and the general metric $\mathcal{A}(z)$ in Euclidean space has the following form:

$$ds^2 = G_{\mu\nu}^s dX^\mu dX^\nu = \frac{h(z)L^2}{z^2} (dt^2 + d\vec{x}^2 + dz^2) \quad (2.2)$$

$$= e^{2\mathcal{A}(z)} (dt^2 + d\vec{x}^2 + dz^2). \quad (2.3)$$

In this context, we introduce the warp factor into the pure AdS₅ to break the conformal symmetry to find the QCD-like gauge theory. We will derive the general formula for heavy quark potential and work out the dilaton potential which will be useful later.

2.1 The heavy quark potential from *AdS/CFT*

To keep the paper self-contained, we follow the standard procedure [29] to derive the static heavy quark potential $V_{Q\bar{Q}}(R)$ under the general metric background of (2.3). In $SU(N)$ gauge theory, the interaction potential for infinity massive heavy quark antiquark is calculated from the Wilson loop

$$W[C] = \frac{1}{N} \text{Tr} P \exp[i \oint_C A_\mu dx^\mu], \quad (2.4)$$

where A_μ is the gauge field, the trace is over the fundamental representation, P stands for path ordering. C denotes a closed loop in spacetime, which is a rectangle with one direction along the time direction of length T and the other space direction of length R . The Wilson loop describes the creation of a $Q\bar{Q}$ pair with distance R at some time $t_0 = 0$ and the annihilation of this pair at time $t = T$. For $T \rightarrow \infty$, the expectation value of the Wilson loop behaves as $\langle W(C) \rangle \propto e^{-TV_{Q\bar{Q}}}$.

According to the *holographic* dictionary, the expectation value of the Wilson loop in four dimensions should be equal to the string partition function on the modified AdS₅ space, with the string world sheet ending on the contour C at the boundary of AdS₅

$$\langle W^{4d}[C] \rangle = Z_{string}^{5d}[C] \simeq e^{-S_{NG}[C]}, \quad (2.5)$$

where S_{NG} is the classical world sheet Nambu-Goto action

$$S_{NG} = \frac{1}{2\pi\sigma_s} \int d^2\eta \sqrt{\text{Det}\chi_{ab}}, \quad (2.6)$$

with σ_s the string tension which has dimension of GeV^{-2} , and χ_{ab} is the induced worldsheet metric with a, b the indices in the $(\eta^0 = t, \eta^1 = x)$ coordinates on the worldsheet. Under the background (2.3), we can obtain the equation of motion:

$$\frac{e^{2\mathcal{A}(z)}}{\sqrt{1 + (z')^2}} = e^{2\mathcal{A}(z_0)}, \quad (2.7)$$

where z_0 is the maximal value of z . Following the standard procedure, one can derive the interquark distance R as a function of z

$$R(z) = 2z \int_0^1 d\nu \frac{e^{2\mathcal{A}(z)}}{e^{2\mathcal{A}(\nu z)}} \frac{1}{\sqrt{1 - \left(\frac{e^{2\mathcal{A}(z)}}{e^{2\mathcal{A}(\nu z)}}\right)^2}}. \quad (2.8)$$

The heavy quark potential can be worked out from the Nambu-Goto string action:

$$V_{Q\bar{Q}}(z) = \frac{1}{\pi\sigma_s} \int_0^1 d\nu e^{2\mathcal{A}(\nu z)} z \frac{1}{\sqrt{1 - \left(\frac{e^{2\mathcal{A}(z)}}{e^{2\mathcal{A}(\nu z)}}\right)^2}}. \quad (2.9)$$

It is noticed that the integral in Eq.(2.9) in principle include some poles, which induces $V_{Q\bar{Q}}(z) \rightarrow \infty$. The infinite energy should be extracted through certain regularization procedure. The divergence of $V_{Q\bar{Q}}(z)$ is related to the vacuum energy for two static quarks. Generally speaking, the vacuum energy of two static quarks will be different in various background. In our latter calculations, we will use the regularized $V_{Q\bar{Q}}^{ren.}$, for example Eqs. (3.3) and (4.4), where the vacuum energy has been subtracted.

2.2 The GKN framework of the gravity dual theory

Motivated from finding the appropriate description of heavy quark potential from gravity theory side, we expect to work out a general classical gravity background. If the deformed AdS_5 metric can describe QCD phenomenology, it is natural to ask whether it is possible to find its dual theory from gravity side. The Gürsoy-Kiritsis-Nitti (GKN) [19] framework offers a systematical procedure to construct the gravity dual theory for a 5D holographic QCD model defined in Eq.(2.3).

In this paper, we follow the notation in Ref.[28] to introduce the GKN framework. According to the GKN's framework, the noncritical string background dual to the QCD-like gauge theories can be described by the following action in the Einstein frame:

$$S_{5D-Gravity} = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-G^E} \left(R - \frac{4}{3} \partial_\mu \phi \partial^\mu \phi - V_B(\phi) \right). \quad (2.10)$$

Where R is the Ricci scalar and has the dimension $[R] = \frac{1}{\text{length}^2}$, κ_5^2 has dimension $[\kappa_5^2] = \text{length}^3$, ϕ is the dilaton field and is dimensionless, $V_B(\phi)$ the dilaton potential and has the dimension of $[V_B(\phi)] = \frac{1}{\text{length}^2}$. The metric in the Einstein frame is denoted by $G_{\mu\nu}^E$ which is related by the metric in the string frame $G_{\mu\nu}^s$ by the following relation:

$$G_{\mu\nu}^E(X) = e^{-\frac{4}{3}\phi} G_{\mu\nu}^s(X). \quad (2.11)$$

In this subsection, the space-time metric has Minkowski signature with the sign convention $(-, +, +, +, +)$.

$$ds_E^2 = e^{2\mathcal{A}(z)} (-dt^2 + d\vec{x}^2 + dz^2), \quad (2.12)$$

here the warp factor $A(z)$ is in the Einstein frame, which is related to $\mathcal{A}(z)$ in the string frame by

$$e^{2A(z)} = e^{-\frac{4}{3}\phi} \frac{h(z)L^2}{z^2} = e^{-\frac{4}{3}\phi} e^{2\mathcal{A}(z)}. \quad (2.13)$$

The two independent Einstein's equations take the following form,

$$3((A'(z))^2 + A''(z)) = -\frac{2}{3}(\phi')^2 - \frac{1}{2}e^{2A(z)}V_B(\phi), \quad (2.14)$$

$$6(A'(z))^2 = \frac{2}{3}(\phi')^2 - \frac{1}{2}e^{2A(z)}V_B(\phi). \quad (2.15)$$

Adding these equations one can obtain a formal expression for the dilaton potential:

$$V_B(\phi(z)) = -e^{-2A(z)}(9(A'(z))^2 + 3A''(z)). \quad (2.16)$$

Subtracting Eq. (2.14) from Eq. (2.15), one can find an important relation between the dilaton and the metric profile:

$$(\phi')^2 = \frac{9}{4}((A'(z))^2 - A''(z)). \quad (2.17)$$

It is noticed that Eq. (2.17) depends on the profile $A(z)$, which is a function of the deformed warp factor $h(z)$ and the dilaton field $\phi(z)$. The resulting second order differential equation for $\phi(z)$ needs two boundary conditions, which we will obtain from the QCD running coupling constant once the bulk coordinate z is connected with the energy scale $E(z)$.

2.3 The running coupling and the beta function

In the GKN framework, the scalar field or dilaton field ϕ encodes the running of the Yang-Mills gauge theory's coupling α . For convenience, the renormalized dilaton field ϕ has been defined as

$$\alpha = e^\phi. \quad (2.18)$$

The warping of the bulk space relates the bulk coordinate z to the energy scale $E(z)$ via the gravitational blue-shift [28]. By using the radial coordinate $r \propto 1/z$, the blue-shift is given by the dimensionless ratio

$$\frac{E_r}{E_{r \rightarrow \infty}} = \sqrt{\frac{G_{tt}(r \rightarrow \infty)}{G_{tt}(r)}}, \quad (2.19)$$

where G_{tt} denotes the temporal component of the metric. In the limit $r \rightarrow \infty$, the space-time is asymptotically flat, and $G_{tt}(r \rightarrow \infty) = -1$. Hence, the blue-shift reads

$$E_{r \rightarrow \infty} = E_r \sqrt{-G_{tt}(r)} \quad \text{or equivalently} \quad E_{r \rightarrow \infty} = E_z \sqrt{-G_{tt}(z)}. \quad (2.20)$$

A simplified expression for the energy scale in the Einstein frame has been given in [28], which has the following form:

$$E_{r \rightarrow \infty} = e^{-\frac{2}{3}\phi(z)} \frac{\sqrt{h(z)}}{z} \quad (2.21)$$

$$= \alpha^{-\frac{2}{3}} \frac{\sqrt{h(z)}}{z} \quad (2.22)$$

If one knows the value of the coupling constant α at a given energy scale $E = E_{r \rightarrow \infty}$, one can find the corresponding value of z from Eq. (2.22). Then at given value of z , one can obtain $\phi(z) = \log(\alpha)$. In order to solve Eq. (2.17), two boundary conditions are needed. In [28], the authors have chosen two points of running coupling $\alpha(E)$ from PDG [30].

The β -function has the definition of

$$\beta \equiv E \frac{d\alpha}{dE}. \quad (2.23)$$

For a 5D holographic model, its β function is related to the deformed warp factor $h(z)$ by

$$\beta \equiv E \frac{d\alpha}{dE} = \frac{e^\phi d\phi}{dA} = \frac{e^{\phi(z)} \cdot \phi'(z)}{A'(z)}. \quad (2.24)$$

As we know, the QCD β -function at 2-loop level has the following form:

$$\beta(\alpha) = -b_0\alpha^2 - b_1\alpha^3, \quad (2.25)$$

with $b_0 = \frac{1}{2\pi}(\frac{11}{3}N_c - \frac{2}{3}N_f)$, and $b_1 = \frac{1}{8\pi^2}(\frac{34}{3}N_c^2 - (\frac{13}{3}N_c - \frac{1}{N_c})N_f)$ [31]. We choose $N_c = 3$ and $N_f = 4$. In this case, $b_0 = \frac{25}{6\pi}$, and $b_1 = \frac{77}{12\pi^2}$.

The yielded beta function in [28] does not monotonically decrease with α . We will show in Sec. III that this behavior of beta function can be improved by choosing different boundary conditions.

3. The *holographic* QCD model with quadratic and logarithmic corrections

In [13], we have derived the $Dp - Dq$ model from top-down method, and found that for any $Dp - Dq$ system except $p = 3$, there is a general logarithmic dilaton background field. In order to generate the Regge behavior for the light flavor mesons, we have proposed the soft-wall Dp-Dq model, which contains a quadratic correction and a logarithmic correction $c_0 \log z$. As pointed in [19], that the logarithmic term $c_0 \log z$ itself cannot produce confinement, while a logarithmic correction with an infrared cut-off in the form of $c_0 \log(z_{IR} - z)$ can have confinement at IR. Therefore,

we extend our soft-wall $Dp - Dq$ model to the following form with the deformed warp factor as

$$h(z) = \exp \left(-\frac{\sigma z^2}{2} - c_0 \ln \left(\frac{z_{IR} - z}{z_{IR}} \right) \right). \quad (3.1)$$

The coefficients σ and c_0 can be either positive or negative. An IR cut-off z_{IR} explicitly sets in the metric, which has the same effect as the hard-wall model [2]. When $c_0 = 0$, $\sigma > 0$ and $\sigma < 0$ corresponds to the soft-wall model [14] and Andreev model, respectively.

In Ref.[21], in order to mimic the QCD running coupling behavior, Pirner and Galow proposed the deformed warp factor

$$h_{PG}(z) = \frac{\log \left(\frac{1}{\epsilon} \right)}{\log \left[\frac{1}{(\Lambda z)^2 + \epsilon} \right]}. \quad (3.2)$$

This metric with asymptotically conformal symmetry in the UV and infrared slavery in the IR region yields a good fit to the heavy $Q\bar{Q}$ -potential with $\Lambda = 264 \text{ MeV}$ and $\epsilon = \Lambda^2 l_s^2 = 0.48$. It is worthy of mentioning that the deformed warp factor $h_{PG}(z)$ is dominated by a quadratic term σz^2 in the UV regime and a logarithmic term $-\log(z_{IR} - z)$ in the IR regime, respectively. The deformed metric in Eq.(3.1) when taking the parameter of $\sigma = 0.08, c_0 = 1, z_{IR} = 2.73 \text{ GeV}^{-1}$ can mimic the Pirner-Galow deformed metric in Eq.(3.2).

Under the background (3.1), the derived heavy quark potential, after subtracted the vacuum energy, has the form of

$$V_{Q\bar{Q}}^{ren.}(z) = -\frac{1}{\pi \sigma_s} \frac{L^2}{z} + \frac{1}{\pi \sigma_s} \frac{L^2}{z} \int_0^1 d\nu \left(\frac{h(\nu z)}{\nu^2} \frac{1}{\sqrt{1 - \nu^4 \left(\frac{h(z)}{h(\nu z)} \right)^2}} - \frac{1}{\nu^2} - \frac{c_0 z}{z_{IR} \nu} \right), \quad (3.3)$$

and the distance between the quark-antiquark R has the form of

$$R(z) = 2z \int_0^1 d\nu \nu^2 \frac{h(z)}{h(\nu z)} \frac{1}{\sqrt{1 - \nu^4 \left(\frac{h(z)}{h(\nu z)} \right)^2}}. \quad (3.4)$$

In the UV limit, i.e, $z \rightarrow 0$, the heavy quark potential has the following simplified expression:

$$\begin{aligned} V_{Q\bar{Q}}^{UV}(R) = & -\frac{0.23 L^2}{\sigma_s R} + \frac{0.17 c_0 L^2}{\sigma_s z_{IR}} \\ & + \frac{(0.22 c_0 + 0.24 c_0^2 - 0.22 \sigma z_{IR}^2) L^2 R}{\sigma_s z_{IR}^2}. \end{aligned} \quad (3.5)$$

It is noticed that the coefficient of the Coulomb part is solely determined by the string tension σ_s , to fit the Cornell potential in UV regime, one can get $\sigma_s = 0.38\text{GeV}^{-2}$. It is also noticed that even in UV limit, both the quadratic and logarithmic corrections contribute to the linear potential. If $c_0 > 0$ and $\sigma > 0$, the contribution to the linear potential from the quadratic term and logarithmic term compete with each other.

3.1 With only quadratic correction

We firstly consider the case with only quadratic correction when $c_0 = 0$.

3.1.1 The heavy quark potential

The heavy quark potential as functions of quark anti-quark distance R for different values of $\sigma = 0.1, 0.01, -0.22, -0.4\text{GeV}^2$ is shown in Fig. 1 (a), and the corresponding relation between R and z is shown in Fig.1 (b). In the numerical calculations, we have chosen the AdS_5 radius $L = 1\text{GeV}^{-1}$, and the Coulomb part is fixed by choosing the string tension $\sigma_s = 0.38$.

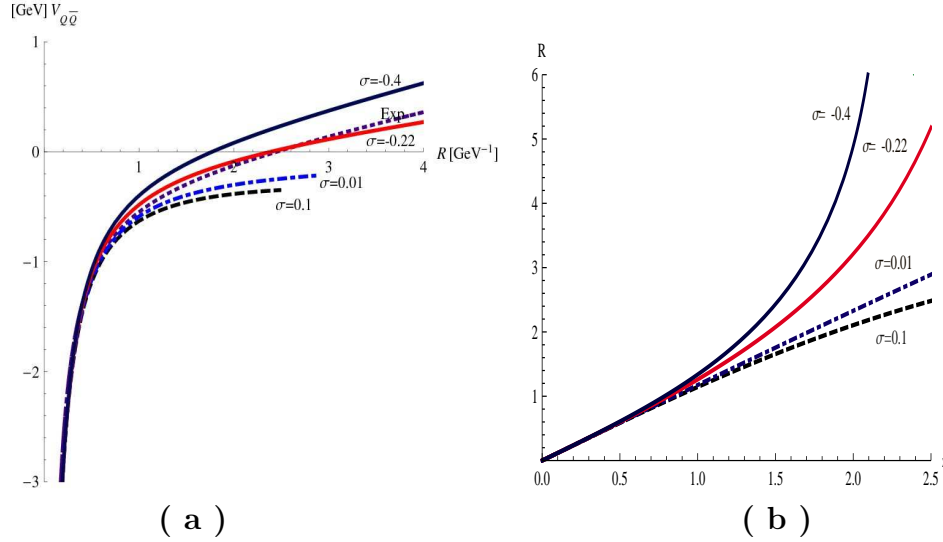


Figure 1: (a) The heavy quark potential as functions of R and (b) the distance R as functions of z in the case of $L = 1\text{GeV}^{-1}$, $\sigma_s = 0.38\text{GeV}^{-2}$, $c_0 = 0$, and $\sigma = 0.1, 0.01, -0.22, -0.4\text{GeV}^2$.

The behavior of $R(z)$ is quite different for $\sigma > 0$ and $\sigma < 0$. In the case of $\sigma > 0$, the interquark distance R firstly increases with z and reach the maximum R_m at certain z_m , then decreases with $z > z_m$. In the case $\sigma < 0$, R diverges at some value of z (this point is defined as z_p), the heavy quark potential also diverges at z_p . When $c_0 = 0$, it can be estimated that $z_p = \sqrt{-2/\sigma}$. Of course, in real QCD system, the quark anti-quark cannot be separated to infinity. From the experimental results, the linear behavior breaks around $R = 1.1\text{fm} = 5.5\text{GeV}^{-1}$, which is about the string breaking scale [32].

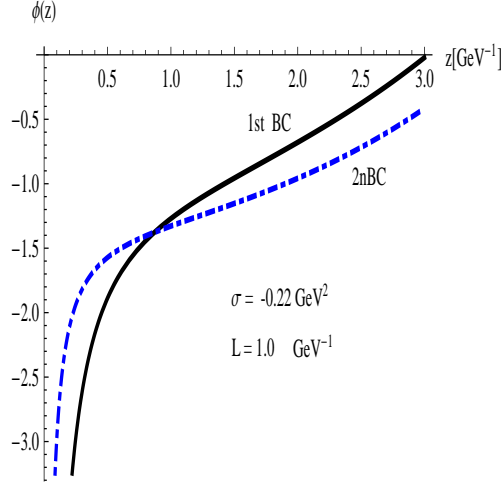


Figure 2: The dilaton field ϕ as a function of the bulk coordinate z in the case of $L = 1\text{GeV}^{-1}$, $\sigma_s = 0.38\text{GeV}^{-2}$, $c_0 = 0$, and $\sigma = -0.22\text{GeV}^2$. The boundary conditions are described in Eq.(3.6).

When $\sigma > 0$, it is found that the quark anti-quark distance R cannot reach the far IR regime. The larger the σ is, the smaller R_m can be reached. The largest R_m for the case of $\sigma > 0$ is around 3GeV^{-1} , which is around 0.8fm . The slope for the linear potential in the middle R regime is found to be much smaller than $\sigma_{str} \approx 0.183$ in the Cornell potential.

When $\sigma < 0$, it is found that the interquark distance R can go to far IR regime. The slope of the linear potential increases with the absolute value of $|\sigma|$. The best fit of the heavy quark potential gives $\sigma = -0.22\text{GeV}^2$. With these parameters, the interquark distance and heavy quark potential diverges at around $z_p = 3.0\text{GeV}^{-1}$. However, it is noticed that in this case, the slope of the linear potential is smaller than the experimental value. For the case of $\sigma = -0.4$, the linear part is parallel to the Cornell potential, however, the value of $V_{Q\bar{Q}}$ is larger than the experimental results. Therefore, strictly speaking, the Cornell potential is not fitted very well in the case with only quadratic correction.

3.1.2 The dilaton potential and the β function for negative σ

From the studies of heavy quark potential, we have found that a negative σ is favored. With the parameters $L = 1\text{GeV}^{-1}$, $\sigma_s = 0.38\text{GeV}^{-2}$, $c_0 = 0$, and $\sigma = -0.22\text{GeV}^2$, the yielded heavy quark potential is close to the Cornell potential. In this subsection, we solve its gravity dual and investigate the β function of this model.

To solve the dilaton field from Eq. (2.17), we need to choose two boundary conditions. For one of the boundary conditions, we use the value of QCD running coupling at 3GeV as input, i.e, $\alpha(E = 3\text{GeV}) = 0.25$, which can be read from [30], and solve z from Eq.(2.22), this gives one boundary condition $\phi(z = 0.87) =$

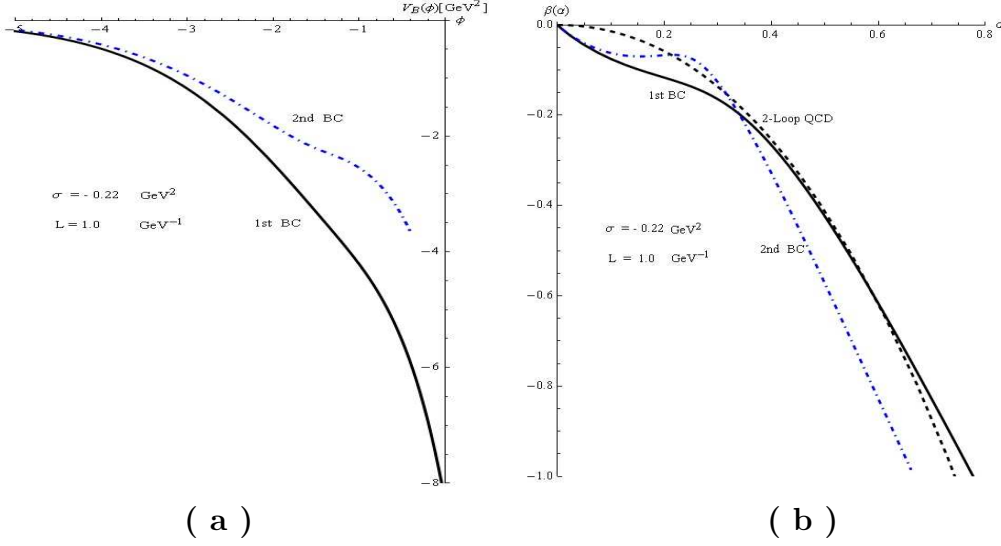


Figure 3: (a) The dilaton potential $V_B(\phi)$ as a function of ϕ and (b) The β function as a function of α in the case of $L = 1\text{GeV}^{-1}$, $\sigma_s = 0.38\text{GeV}^{-2}$, $c_0 = 0$, and $\sigma = -0.22\text{GeV}^2$. The boundary conditions are described in Eq.(3.6).

$\log(0.25)$. It is noticed that 3GeV is about the charmonium mass which is in the IR region.

For another boundary condition, we can choose the same boundary condition as in [28] by input the running coupling at 8GeV , which gives the boundary condition $\alpha(8\text{GeV}) = 0.18575$. However, because the produced β function in [28] is not a monotonic function, we guess this strange behavior is induced by fixing two points of the running coupling. Therefore, we choose to use the derivative of the dilaton field at $z(E = 3\text{GeV}) = 0.87$, i.e, $\phi'(z = 0.87)$ as another boundary condition. Because we don't know the value of the $\phi'(z = 0.8701)$, we choose it as a free parameter.

There are two types of boundary conditions we used:

$$\begin{aligned} \text{1stBC} : \phi(z = 0.87) &= \log(0.25), \quad \phi'(z = 0.87) = 0.9, \\ \text{2ndBC} : \phi(z = 0.87) &= \log(0.25), \quad \phi(z = 0.38) = \log(0.18). \end{aligned} \quad (3.6)$$

where $\phi'(z = 0.87) = 0.9$ is used by the best fit of the QCD β function, and $\phi(z = 0.38) = \log(0.18)$ is from the input of running coupling $\alpha(8\text{GeV}) = 0.18$ at UV.

The dilaton field ϕ as a function of z is shown in Fig. 2. It is found that for the two types of boundary conditions, the solution of the dilaton field $\phi(z)$ is monotonically increasing with z . For the 1st type of boundary condition, ϕ increases more quickly with z than the case with 2nd type boundary condition.

The dilaton potential $V_B(\phi)$ as a function of ϕ and the β function as a function of α are shown in Fig. 3 (a) and (b), respectively. It is found that for both types of boundary conditions, $V_B(\phi)$ decreases with ϕ , the dilaton potential in the IR regime

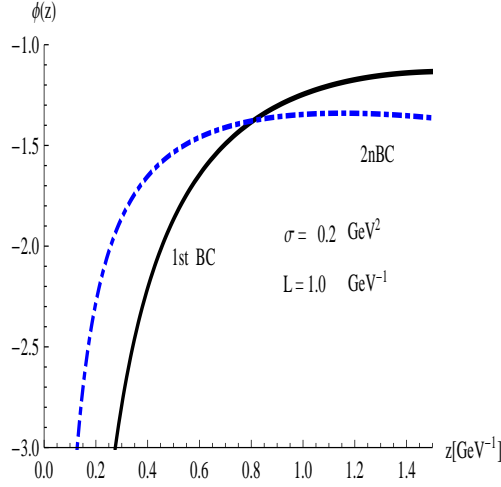


Figure 4: The dilaton field ϕ as a function of the the bulk coordinate z in the case of $L = 1\text{GeV}^{-1}$, $\sigma_s = 0.38\text{GeV}^{-2}$, $c_0 = 0$, and $\sigma = 0.22\text{GeV}^2$. The boundary conditions are described in Eq.(3.7).

is not bounded from below, which might indicate a unstable vacuum. For the second type boundary condition, i.e, the boundary condition used in [28], it is found that the produced β function is not a monotonic function of coupling α . This behavior as we have discussed, is due to the fixing running coupling constant at two points. For the first type of boundary condition, the produced β function is monotonically decreasing with the coupling constant α , and it agrees reasonably well with the QCD β function, which is shown by dashed line in Fig. 3(b).

3.1.3 The dilaton potential and the β function for positive σ

As a reference, we solve the gravity dual and investigate the β function for the case of positive σ , which corresponds to the KKSS model or soft-wall model. The parameters for the model are $L = 1\text{GeV}^{-1}$, $\sigma_s = 0.38\text{GeV}^{-2}$, $c_0 = 0$, and $\sigma = 0.2\text{GeV}^2$.

The two types of boundary conditions are

$$\begin{aligned} \text{1stBC} : \phi(z = 0.81) &= \log(0.25), \quad \phi'(z = 0.81) = 0.9, \\ \text{2ndBC} : \phi(z = 0.81) &= \log(0.25), \quad \phi(z = 0.38) = \log(0.18), \end{aligned} \quad (3.7)$$

which are almost the same as Eq.(3.6).

The dilaton field ϕ as a function of z is shown in Fig. 4. It is found that for the two types of boundary conditions, the solution of the dilaton field $\phi(z)$ monotonically increases to a maximum value at z_m . For the first type boundary condition, $z_m = 1.4\text{GeV}^{-1}$, and for the 2nd type boundary condition, $z_m = 1.0\text{GeV}^{-1}$. For both cases, z_m is much smaller than z_{IR} .

The dilaton potential $V_B(\phi)$ as a function of ϕ and the β function as a function of α are shown in Fig. 5 (a) and (b), respectively. The dilaton potential $V_B(\phi)$

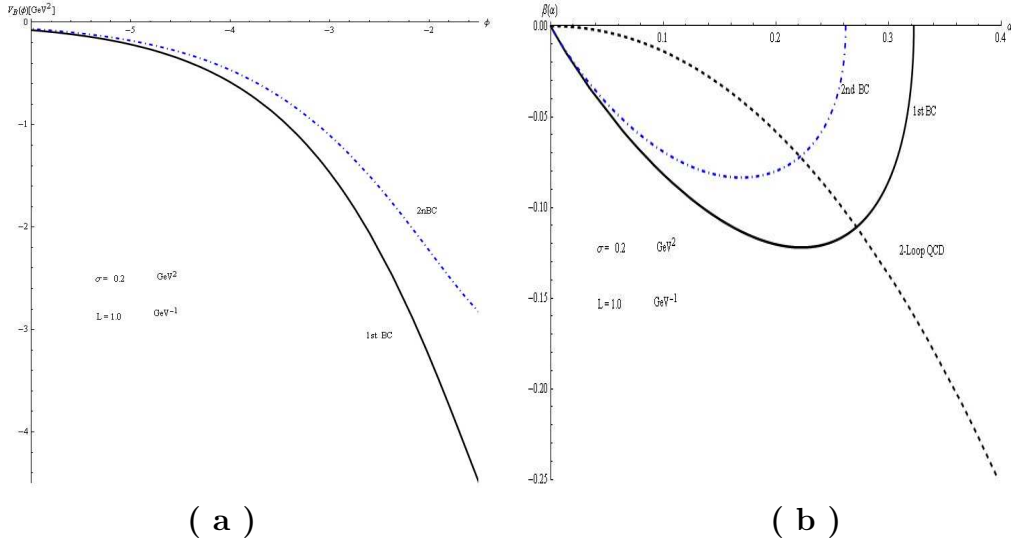


Figure 5: (a) The dilaton potential $V_B(\phi)$ as a function of ϕ and (b) The β function as a function of α in the case of $L = 1\text{GeV}^{-1}$, $\sigma_s = 0.38\text{GeV}^{-2}$, $c_0 = 0$, and $\sigma = 0.2\text{GeV}^2$. The boundary conditions are described in Eq.(3.7).

decreases with ϕ , which shows an unstable potential. It is found that the β function is very interesting in the case of a positive σ . The β function has two fixed points where β function vanishes: 1) one is a UV fixed point, where $\beta = 0$ when $\alpha = 0$, 2) another is a IR fixed point, where $\beta = 0$ at a moderate strong coupling constant $\alpha = 0.26$ for 2ndBC and $\alpha = 0.32$ for 1stBC, respectively. If we take a larger value of σ , the IR fixed point will appear at a smaller coupling constant, then we can have the Banks-Zaks fixed point [33].

3.2 With both quadratic and logarithmic corrections

3.2.1 The heavy quark potential

From the Pirner-Galow metric Eq.(3.2), we extract the coefficient of $c_0 = 1$. Fig. 6 (a) shows the heavy quark potential in the case of $c_0 = 1$, the best fitted result (black solid line) gives $\sigma = 0.34\text{GeV}^2$ and $z_{IR} = 2.54\text{GeV}^{-1}$. With these parameters, the interquark distance and the heavy quark potential diverges at $z_p = 1.95\text{GeV}^{-1}$. It is found that the heavy quark potential is perfectly fitted in the regime $R < 0.5\text{GeV}^{-1}$ and $R > 2\text{GeV}^{-1}$, however, in the regime $0.5\text{GeV}^{-1} < R < 2\text{GeV}^{-1}$, the fitted heavy quark potential is a little bit higher than the experimental data.

The result from the Pirner-Galow model is also shown in the figure by using the short dashed line. It is found that the Coulomb part is in good agreement with the Cornell potential, the linear part is parallel to the Cornell potential, however, the value of $V_{Q\bar{Q}}$ is a little bit higher than the data.

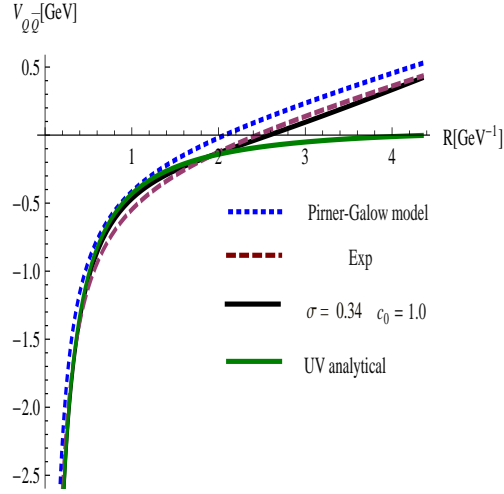


Figure 6: (a) The best fitted heavy quark potential as a function of R in the case of $c_0 = 1$ compared with Pirner-Galow result, the UV analytical result and the Cornell potential. In the case of $c_0 = 1$, the other parameters are $L = 1\text{GeV}^{-1}$, $\sigma_s = 0.38\text{GeV}^{-2}$, $\sigma = 0.34\text{GeV}^2$ and $z_{IR} = 2.54\text{GeV}^{-1}$.

The green solid line is the UV analytical result from Eq.(3.5). It is found that this result is in good agreement with the Coulomb part of the Cornell potential in the region $R < 2\text{GeV}^{-1}$. The UV analytical is not valid any more above 2GeV^{-1} .

3.2.2 The dilaton potential and the β function

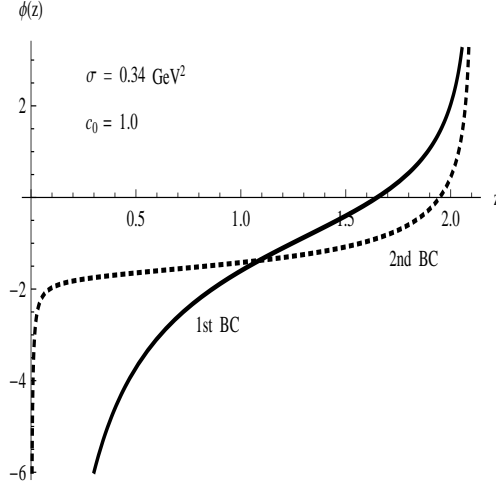


Figure 7: The dilaton field ϕ as a function of the the bulk coordinate z in the case of $c_0 = 1$, $L = 1\text{GeV}^{-1}$, $\sigma_s = 0.38\text{GeV}^{-2}$, $\sigma = 0.34\text{GeV}^2$ and $z_{IR} = 2.54\text{GeV}^{-1}$. The two types of boundary conditions are described in Eq.(3.8).

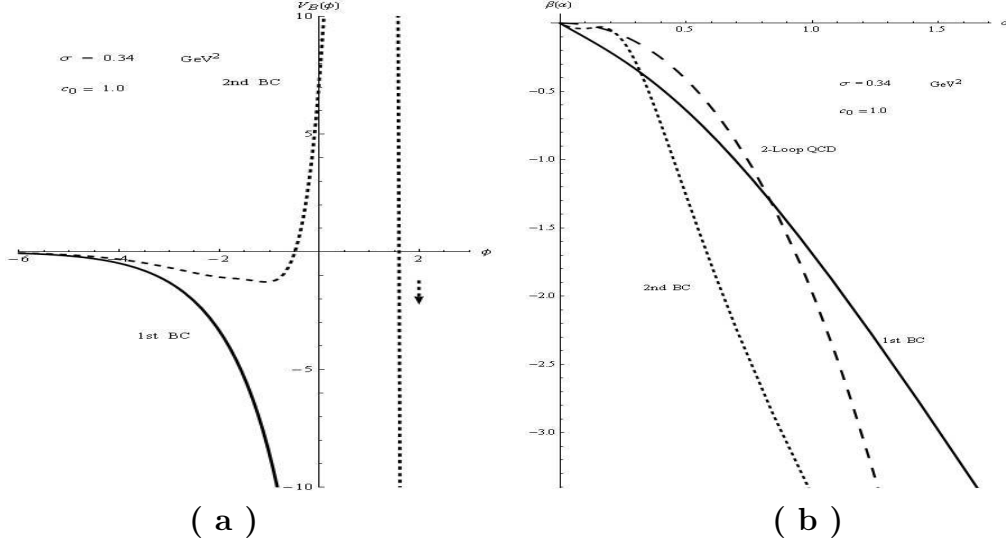


Figure 8: (a) The dilaton potential $V_B(\phi)$ as a function of ϕ and (b) the β function as a function of coupling constant α in the case of $c_0 = 1$, $L = 1\text{GeV}^{-1}$, $\sigma_s = 0.38\text{GeV}^{-2}$, $\sigma = 0.34\text{GeV}^2$ and $z_{IR} = 2.54\text{GeV}^{-1}$. The two types of boundary conditions are described in Eq.(3.8).

The dilaton field ϕ as a function of z is shown in Fig. 7 for two different type of boundary conditions:

$$\begin{aligned} \text{1stBC} : \phi(z = 1.08) &= \log(0.25), \quad \phi'(z = 1.08) = 2.5, \\ \text{2ndBC} : \phi(z = 1.08) &= \log(0.25), \quad \phi(z = 0.42) = \log(0.18). \end{aligned} \quad (3.8)$$

Where $\phi(z = 1.08) = \log(0.25)$ is from the input of running coupling $\alpha(E = 3\text{GeV}) = 0.25$ at IR, $\phi'(z = 1.08) = 2.5$ is by choosing the best fit of the β function, and $\phi(z = 0.42) = \log(0.18)$ is from the input of QCD running coupling $\alpha(E = 8\text{GeV}) = 0.18$ at UV.

It is found that for these two types of boundary conditions, the solution of the dilaton field $\phi(z)$ increases monotonically with z . The difference lies in that ϕ is flat in a rather wide region of z for 2nd type of boundary condition.

The dilaton potential $V_B(\phi)$ as a function of ϕ and the β function as a function of α are shown in Fig. 8 (a) and (b), respectively. It is found that for the first type boundary condition, $V_B(\phi)$ is not bounded from below in the IR region, however, the β function monotonically decreases with the increase of α , which qualitatively agrees with the behavior of QCD β function. For the second type boundary condition, it is found that the dilaton potential $V_B(\phi)$ is unstable in the IR, however, the produced β function is not a monotonic function of coupling α . This behavior as we have pointed out in Sec.2.3, is due to the fixing of two points of running coupling constant.

3.3 With only logarithmic correction

3.3.1 The heavy quark potential

We now consider the case with only logarithmic correction when $\sigma = 0$. The best fitted heavy quark potential as functions of quark anti-quark distance R is shown in Fig. 9 (a) by using the black solid line. The results are compared with that from the Pirner-Galow model (short dashed line) and the experimental data (the long dashed line) and the UV analytical result. The best fit of the heavy quark potential gives $c_0 = 0.272\text{GeV}^2$ and $z_{IR} = 2.1\text{GeV}^{-1}$. With these parameters, numerical calculations shows that the interquark distance R becomes divergent at $z_p = 1.85\text{GeV}^{-1}$ (rough estimates gives $z_p \sim \frac{2z_{IR}}{c_0+2}$). It is found that the heavy quark potential can be perfectly fitted in the whole regime of R with only logarithmic correction.

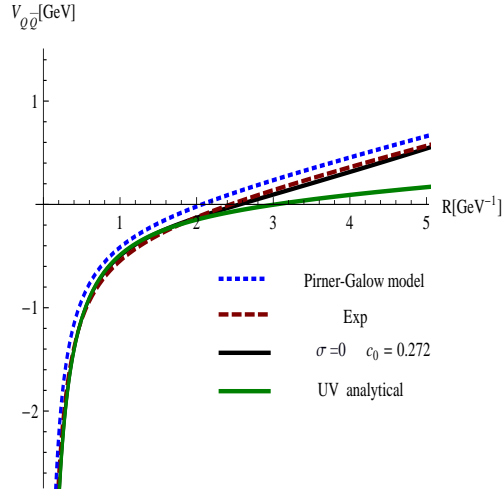


Figure 9: (a) The heavy quark potential as functions of the distance R in the case of $L = 1\text{GeV}^{-1}$, $\sigma_s = 0.38\text{GeV}^{-2}$, $\sigma = 0$ and $c_0 = 0.272$ and $z_{IR} = 2.1\text{GeV}^{-1}$.

3.3.2 The dilaton potential and the β function

The dilaton field ϕ as a function of z is shown in Fig. 10 for two different type of boundary conditions:

$$\begin{aligned} \text{1stBC : } \phi(z = 0.9) &= \log(0.25), \quad \phi'(z = 0.9) = 1.7, \\ \text{2ndBC : } \phi(z = 0.9) &= \log(0.25), \quad \phi(z = 0.39) = \log(0.185). \end{aligned} \quad (3.9)$$

Where $\phi(z = 0.9) = \log(0.25)$ is from the input of the $\alpha(E = 3\text{GeV}) = 0.25$ at IR, $\phi'(z = 0.9) = 1.7$ is determined by choosing the best fitting of QCD β function, and $\phi(z = 0.39) = \log(0.18)$ is from $\alpha(E = 8\text{GeV}) = 0.18$ at UV.

It is found that for these two types of boundary conditions, the solution of the dilaton field $\phi(z)$ increases monotonically with z . The difference lies in that ϕ is flat in a rather wide region of z for 2nd type of boundary condition.

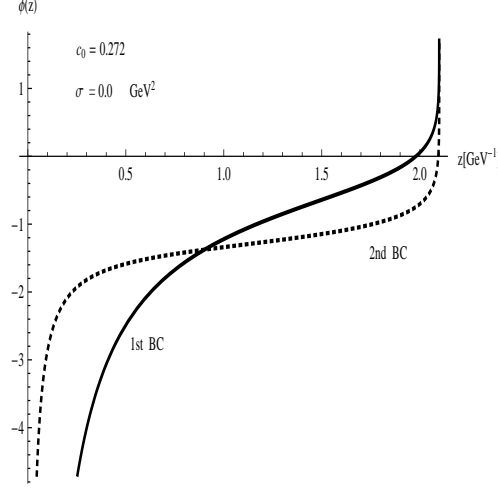


Figure 10: The dilaton field ϕ as a function of z in the case of $L = 1\text{GeV}^{-1}$, $\sigma_s = 0.38\text{GeV}^{-2}$, $\sigma = 0$ and $c_0 = 0.272$ and $z_{IR} = 2.1\text{GeV}^{-1}$. The boundary conditions are described in Eq.(3.9).

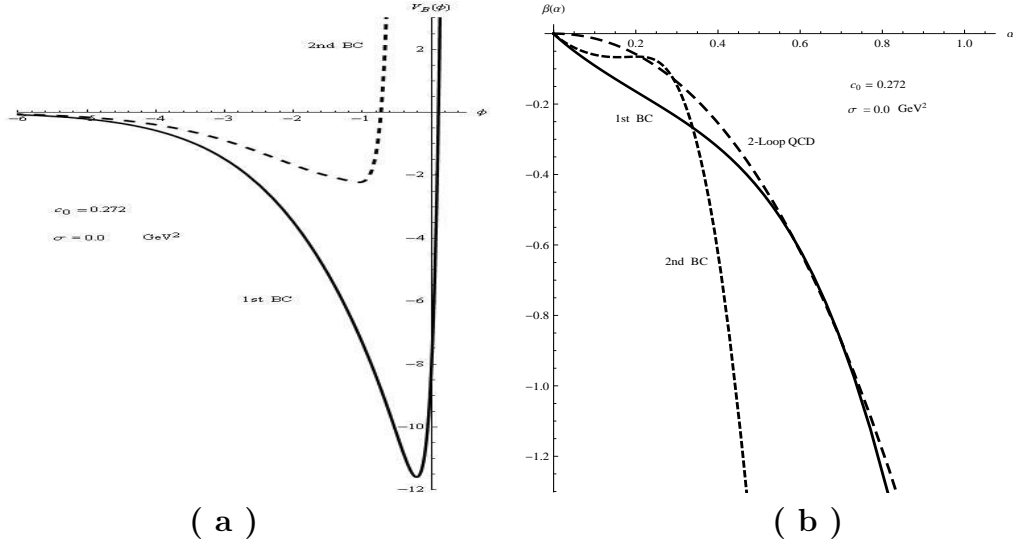


Figure 11: (a) The dilaton potential V_B as a function of ϕ and (b) the β function as a function of coupling constant α in the case of $L = 1\text{GeV}^{-1}$, $\sigma_s = 0.38\text{GeV}^{-2}$, $\sigma = 0$ and $c_0 = 0.272$ and $z_{IR} = 2.1\text{GeV}^{-1}$. The boundary conditions are described in Eq.(3.9).

The dilaton potential $V_B(\phi)$ as a function of ϕ and the β function as a function of α are shown in Fig. 11 (a) and (b), respectively. It is found that for the second type boundary condition, the dilaton potential $V_B(\phi)$ is stable which is bounded from below in the IR, however, the produced β function is not a monotonic function of coupling α . This behavior as we have discussed, is due to the fixing of two points of running coupling constant. For the first type boundary condition, $V_B(\phi)$ is also a

stable potential which is more deeply bounded from below in the IR region, moreover, the β function monotonically decreases with the increase of α , and coincides with the QCD β function in IR regime.

It is noticed that in the case with only logarithmic correction, the 5D dilaton potential $V_B(\phi)$ has the same shape as the dilaton potential in an effective 4D QCD model in [34].

4. The more compact model with only logarithmic corrections

From studies in previous section, it is found that the model with only logarithmic correction in the deformed warp factor can fit the heavy quark potential perfectly, which is much better than the model with only quadratic correction. It might not be a surprise because there are four parameters used, i.e, the deformed AdS_5 radius L , the string tension α , the coefficient c_0 and the IR cut-off z_{IR} , while for the model with only quadratic correction, there are only three parameters, i.e, the deformed AdS_5 radius L , the string tension σ_s , and the coefficient σ .

It should be mentioned that for the case of $c_0 = 1$ in Sec. 3.2, five parameters have been used to fit the heavy quark potential, and the best fitted result is better than the model with only quadratic correction, but not as good as the model with only logarithmic correction. Remind of the results in Ref. [27], White found that the model with only quadratic correction, which has less parameters are better than the backreaction model, which has more parameters to produce the heavy quark potential. Therefore, it is not necessarily correct that one can fit the three parameters in the Cornell potential with enough parameters.

Still, we hope to improve our model with only logarithmic correction. It is found there are two length scales in the model, i.e, the deformed AdS_5 radius L and the IR cut-off z_{IR} . We can combine these two length scales into one, and choose the following metric structure:

$$\begin{aligned} ds^2 &= G_{\mu\nu} dX^\mu dX^\nu = e^{2\mathcal{A}(z)} (dt^2 + d\vec{x}^2 + dz^2) \\ &= \frac{h(z)z_{IR}^2}{z^2} (dt^2 + d\vec{x}^2 + dz^2) \end{aligned} \quad (4.1)$$

with $\mathcal{A}(z)$ and $h(z)$ taking the following expressions:

$$\mathcal{A}(z) = -\log\left(\frac{z}{z_{IR}}\right) - \frac{c_0}{2} \log\left(\frac{z_{IR} - z}{z_{IR}}\right) \quad (4.2)$$

$$h(z) = \exp\left(-c_0 \log\left(\frac{z_{IR} - z}{z_{IR}}\right)\right) \quad (4.3)$$

Following the same procedure, we obtain the expression of the renormalized heavy quark potential $V_{Q\bar{Q}}^{ren.}$ in the form of

$$V_{Q\bar{Q}}^{ren.}(z) = -\frac{1}{\pi\sigma_s} \frac{z_{IR}^2}{z} + \frac{1}{\pi\sigma_s} \frac{z_{IR}^2}{z} \int_0^1 d\nu \left(\frac{h(\nu z)}{\nu^2} \frac{1}{\sqrt{1 - \nu^4 \left(\frac{h(z)}{h(\nu z)} \right)^2}} - \frac{1}{\nu^2} - \frac{c_0 z}{z_{IR} \nu} \right) \quad (4.4)$$

and the interquark distance R has the form of

$$R(z) = 2z \int_0^1 d\nu \frac{e^{2\mathcal{A}(z)}}{e^{2\mathcal{A}(\nu z)}} \frac{1}{\sqrt{1 - \left(\frac{e^{2\mathcal{A}(z)}}{e^{2\mathcal{A}(\nu z)}} \right)^2}}. \quad (4.5)$$

The UV limit of the heavy quark potential has expression of

$$V_{Q\bar{Q}}(R) = -\frac{0.23z_{IR}^2}{\sigma_s R} + \frac{0.17c_0 z_{IR}}{\sigma_s} + \frac{(0.22c_0 + 0.24c_0^2) R}{\sigma_s}. \quad (4.6)$$

The Coulomb part can be fitted very well with the string tension $\sigma_s = 1.6\text{GeV}^{-2}$, which is just z_{IR}^2/L^2 times $\sigma_s = 0.38$ in Sec.3.3.1. The best fit of the heavy quark potential gives $c_0 = 0.272\text{GeV}^2$ and $z_{IR} = 2.11\text{GeV}^{-1}$, which are the same as those in Sec.3.3.1. The results of $\phi(z)$, $V_B(\phi)$ and $\beta(\alpha)$ in the compact model are almost the same as those in Sec. 3.3.2, therefore, we neglect the figures in this part.

The advantage of the compact model is that with only three parameters, we can produce the results of heavy quark potential and QCD β function as good as those in the model with four parameters.

5. Conclusion and discussion

In this paper, we study a *holographic* QCD model which contains a quadratic term $-\sigma z^2$ and a logarithmic term $-c_0 \log[(z_{IR} - z)/z_{IR}]$ with an explicit infrared cut-off z_{IR} in the deformed AdS_5 warp factor. We investigate the heavy quark potential, solve the dual gravity with dilaton field in Gürsoy -Kiritsis-Nitti (GKN) framework, and study the corresponding β function for three cases, i.e, with only quadratic correction, with both quadratic and logarithmic corrections and with only logarithmic correction. Our studies show that in the case with only quadratic correction, the heavy quark potential can be qualitatively fitted with a negative σ , and the beta-function agrees with the QCD beta-function reasonably well, however, the dilaton potential is unbounded in infrared regime. In the case with only logarithmic correction, the heavy quark Cornell potential can be fitted very well, the corresponding beta-function agrees with the QCD beta-function at 2-loop level reasonably well, and the dilaton potential is bounded from below in infrared. We also propose a more

compact model which has only logarithmic correction in the deformed warp factor, which can describe the heavy quark potential and QCD β function very well with only three parameters.

Stability analysis of the dilaton potential V_B

From our numerical studies, it is shown that the dilaton potential $V_B(\phi)$ for the case with only quadratic correction keeps decreasing with ϕ , which indicates the potential is unstable. For the case with only logarithmic correction, the dilaton potential firstly decreases with ϕ then moves upward in the IR regime, which indicates that the dilaton potential is stable. In the following, we analyze the stability of the dilaton potential for the given metric Eq.(3.1).

Because ϕ monotonically increases with z , we can analyze the stability of $V_B(\phi)$ from $V_B(z)$ in Eq.(2.16). Substitute Eq.(2.17) into Eq.(2.16), we can have the expression as:

$$V_B(z) = -e^{-2A(z)} \left(12(A'(z))^2 - \frac{4}{3}(\phi'(z))^2 \right). \quad (5.1)$$

For the metric structure Eq.(3.1) in the string frame with quadratic and logarithmic correction, the metric in the Einstein frame takes the explicit form of

$$A(z) = -\frac{2}{3}\phi - \mathcal{A}(z), \quad \mathcal{A}(z) = \log z + \frac{1}{4}\sigma z^2 + \frac{c_0}{2} \log \frac{z_{IR} - z}{z_{IR}}, \quad (5.2)$$

and the dilaton potential Eq.(5.1) becomes

$$V_B(z) = -4e^{\frac{4}{3}\phi + 2\mathcal{A}} [(\phi')^2 + 4\phi'\mathcal{A}' + 3(\mathcal{A}')^2]. \quad (5.3)$$

Because the two square terms in the bracket are always positive, also from our numerical results for the physical cases, we have $\phi' > 0$, the only chance for $V_B(z)$ to change sign in the IR regime is to have a negative derivative of \mathcal{A} . From the explicit expression of \mathcal{A}' , i.e.,

$$\mathcal{A}' = \frac{1}{z} + \frac{\sigma z}{2} - \frac{c_0}{2(z_{IR} - z)}, \quad (5.4)$$

we can read that with only quadratic correction, i.e., when $c_0 = 0$, if $\sigma > 0$, \mathcal{A}' is always positive. When $\sigma < 0$, e.g., $\sigma = -0.22$ to produce the Cornell potential, \mathcal{A}' is also positive in the regime of $z < z_{IR}$. In the case with only logarithmic correction, i.e., when $\sigma = 0$, we can see that in the IR regime when $z \rightarrow z_{IR}$, $\mathcal{A}' \rightarrow -\infty$ when $c_0 > 0$, therefore, $V_B(z)$ might change sign and become positive in the IR regime.

Positive or negative quadratic correction? To fit the heavy quark potential and to produce the QCD β function, a negative σ , i.e., the Andreev model is favored. However, from our previous experience in Ref.[13], to produce the Regge behavior of ρ meson, a positive σ is needed, i.e., the KKSS model or soft-wall model is favored. One possible explanation is that different model is needed to describe the physics in light flavor sector and heavy flavor sector, respectively. The subtlety of the quadratic correction in the *holographic* model deserves further careful studies in the future. In

our future project, we will check whether Regge behavior can be described in the holographic model with only logarithmic correction.

One interesting result in the KKSS model is that the corresponding β function exists a IR fixed point. If the fixed point exists at a strong coupling regime, the KKSS model might be interesting to study the unitary regime of BCS-BEC crossover in cold atom system. If the fixed point exists at a weak coupling regime, which might be interesting to study the dynamical electroweak symmetry breaking physics.

Acknowledgments:

The authors thank B.Galow, P. Hohler, F. Jugeau, M.S. Ma, S. Pu, M. Stephanov, N. Su, F.K. Xu, Y. Yang, and H.Q. Zhang for valuable discussions. The work of M.H. is supported by CAS program "Outstanding young scientists abroad brought-in", CAS key project KJCX3-SYW-N2, NSFC10735040, NSFC10875134, and K.C.Wong Education Foundation, Hong Kong.

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